WMPLS THROUGHPUT EFFICIENCY IN MULTIPATH RAYLEIGH FADING ENVIRONMENTS

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Abstract—*This paper studies the throughput efficiency of Wireless Multi-Protocol Label Switching (WMPLS) applying data link Automatic Repeat reQuest (ARQ), Go-Back-N (GBN) and Selective Repeat(SR) protocols, for reliable packet transmission. The paper investigates the impact of the packet size on the throughput efficiency of the ARQ protocol for a reliable service under different conditions of a multipath fading channel. This investigation addresses the probability of receiving a correct packet. An upper bound and a lower bound for this probability are suggested in the closed-form format. A novel numerical method to calculate the optimal packet size for GBN is also proposed in this paper.*

Keywords: *WMPLS, Packet Size, and Multipath Fading Channel Modeling.*

I. INTRODUCTION

Multi-Protocol Label Switching (MPLS) [1] is a packet forwarding approach that provides architecture for aggregating packets into Forwarding Equivalency Classes (FEC) and forwarding the packets over established paths called label switching paths (LSP). These paths need to be established before any packets are sent. MPLS uses a technique called label switching to forward the packet over LSP. This is accomplished by encoding a fixed-length label in the packet header to find the next hop along the path. Each router along the path looks at the incoming label to decide on the outgoing interface and the outgoing label. The intermediate routers along the LSP swap the incoming label with the outgoing label and switch the packet to the outgoing interface. The last router along the LSP strips off the label and forwards the packet based on its destination IP address. MPLS is a technology that not only provides speedy process to improve the existing IP performance, but also supports Quality of Service (QoS). These features motivate to extend MPLS to be employed in wireless networks. Wireless MPLS networks can be connected with a wire line MPLS network to provide seamless connectivity. Wireless MPLS [2] uses open radio waves instead of a cable; therefore, implementing MPLS in wireless networks requires necessary extensions to count for the nature of the media. The extensions include changes to the label format; extensions to the two available signaling protocols (RSVP-TE [3] and CR-LDP [4]); and, employment of different mechanisms for LSP setup in Mobile Ad Hoc networks, cellular networks and Mobile IP networks. The purpose of this paper is to study the Automatic Repeat reQuest (ARQ) protocols performance in using WMPLS over fading wireless data links. The results of the study have shown that using WMPLS over wireless data links can be satisfactory if the optimum packet length is chosen as well as ARQ protocol to ensure that the wireless data link will not degrade the WMPLS operation. Wireless data links are different from wired data links, since the Bit Error Rate (BER) in wireless links is much higher. The impact of these characteristics is investigated in Section II; multipath Rayleigh fading channel is explained in Section III; performance analysis of ARQ protocols as well as obtained results and related discussion are presented in Section IV; and, conclusions are drawn in Section V.

II. OPERATION OF ARQ PROTOCOLS OVER WIRELESS LINKS

Reliable data link protocols use the ARQ window protocols mechanisms (i.e., Go-Back-N (GBN) and Selective Repeat (SR)) to ensure that the transmitted data is received without errors at its destination. In GBN ARQ protocol the transmitter is allowed to transmit multiple frames without waiting for acknowledgment. There is a constraint on the number of frames sent without acknowledgments. This number should not exceed some maximum allowable number, N, of an unacknowledged frames. When a packet is lost or corrupted the protocol requires the retransmission of any lost or corrupted packets and all the subsequent packets sent prior to the discovery of the lost or corrupted packet. Therefore, when the window size is large and the channel has a high bit probability error, a single frame error can cause GBN to retransmit a large number of frames. However, The SR protocol avoids unnecessary retransmissions by having the transmitter retransmits only the frames that are lost or corrupted. A window size of N is still used as in GBN and a number of frames up to N are allowed to be transmitted without Acknowledgment; however, unlike GBN the SR will acknowledge a correctly received packet whether or not it is in order. Out-of-order frames are buffered until any missing frames are received. Therefore, SR is more complex than GBN. The metric used to evaluate the performance of these two protocols is the throughput efficiency. The effect of fading on the throughput efficiency is investigated in more details in the following sections and an optimum packet length calculation is suggested to improve the protocols' efficiency.

CCECE 2004- CCGEI 2004, Niagara Falls, May/mai 2004
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III. MULTIPATH FADING CHANNEL MODELING

In this paper, we consider a flat Rayleigh fading channel based on the Clarke's model [5], [6]. Clarke developed a model assuming a field comprised of many azimuthal plane waves with arbitrary carrier phases, arbitrary azimuthal angles of arrival, and equal average amplitudes per wave. Waves incident on the mobile undergo a Doppler shift as well due to the motion of the receiver and arrive at the receiver at the same time. Under these assumptions, a Rayleigh distributed envelope for the received signal is formed [6]. The Rayleigh distribution is commonly used in mobile radio channels to describe the statistical time varying nature of the received envelope of a flat fading signal:

$$f(r) = \frac{r}{\sigma^2} \exp\left(\frac{r^2}{2\sigma^2}\right), \quad r \geqslant 0 \tag{1}$$

where r is the complex envelope of the channel gain, $f(r)$ is the probability density function (pdf), and σ^2 is the variance of the channel response. In other words, σ^2 contains the effect of any source that has impact on the correlation of the received signal as space, time, or frequency. For example, the Jake's model proposed in [6] takes into account the temporal variations of the propagation environment (Doppler effect).

IV. PERFORMANCE ANALYSIS OF ARQ PROTOCOLS

IV-A. Mathematical Formulation

As mentioned in the previous section, the GBN protocol is less efficient than the SR mechanism, since the former requires the retransmission of all packets in the transmission window. The throughput efficiency of GBN is defined in (2) as a direct result of the analysis of Cain [7], [8] who studied the packet length size in ATM wireless networks. The study is extended to WMPLS. WMPLS packets differ from ATM cells by being of variable length. The following equations define the throughput efficiency, η, in the WMPLS networks [7], [8]:

$$\eta = (1 - R_{\text{OHD}}) \cdot \frac{P_C}{P_C + (1 - P_C)N_{\text{WIN}}} \tag{2a}$$

where,

$$\begin{aligned} R_{\text{OHD}} &\triangleq \frac{\text{packet} - \text{overhead}}{\text{packet} - \text{overhead} + \text{packet} - \text{payload}} \\ &= \frac{h}{h+T}. \end{aligned} \tag{2b}$$

h is the number of overhead bits per packet, T is the number of information bits per packet, N_{WIN} is the transmission window size in packets, and P_C is the probability of correctly receiving a packet. The probability P_C depends on the communication system and the channel which is usually modeled by BER. The BER depends on the physical parameters, the mobile speed, and thermal noise of the environment [5]. This dependency is studied in the previous section. To correctly receive a packet, overhead and payload should be received without errors. Any bit of error will result in corrupted packet. Therefore, the probability to get a correct packet is

$$P_C \triangleq (1-p)^{h+T} \tag{3}$$

where p is the BER of the channel. The BER is usually a direct function of the signal-to-noise ratio (SNR). For example for a non-fading channel (i.e., when the signal envelope r is fixed), the BER of binary PSK as a function of the received SNR γ is [9]

$$p(\gamma) = Q(\sqrt{2\gamma}) \tag{4}$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}dt}$ is called the Q-function, $\gamma \triangleq r^2 \varepsilon_b / N_0$, ε_b is the transmitted energy per bit, and $N_0 \triangleq \varsigma^2/2$ is the additive white Gaussian noise power (ς^2 is the variance of the additive noise). Either numerical or analytical BER calculation for other modulation schemes is easily legible, e.g., [9]. All results of analysis in this paper are based on binary PSK as the modulation scheme.

We view (4) as a conditional error probability with the condition fixed r. In a fast fading environment the channel varies between transmission of different bits of the packet. To obtain the probability of correct packet P_{C} in this situation, we plug (4) in (6), then take the expectation of the result with respect to r which is Rayleigh distributed as follows[1]:

$$\begin{aligned} P_C &= E_\gamma \left[\left(1 - Q(\sqrt{2\gamma})\right)^{h+T} \right] && \text{(6a)} \\ &= \frac{1}{\overline{\gamma}} \int_0^\infty \left(1 - Q(\sqrt{2\gamma})\right)^{h+T} e^{-\gamma/\overline{\gamma}} d\gamma && \text{(6b)} \end{aligned}$$

where $\overline{\gamma} = \left(2 - \frac{\pi}{2}\right) \sigma^2 \varepsilon_b / N_0$. The exact result of (6) is possible by numerical computation. Here, we give an upper bound and a lower bound for the probability of correct packet. Using Jensen's inequality[2] [12] applied to the convex function $f(X) = (1 - X)^{h+T}$ for the lower bound, and using the approximation $(1 - x)^n \approx 1 - nx$ for very small values of x as well as a closed-form integration of Q-function[3], we get the following:

$$\left(1 - \frac{1}{2}\left(1 - \left[\frac{\overline{\gamma}}{1+\overline{\gamma}}\right]^{1/2}\right)\right)^{h+T} \leqslant P_C \tag{7a}$$

$$P_C \leqslant 1 - \frac{(h+T)/2}{\sqrt{1+\overline{\overline{\gamma}}}\left(\sqrt{\overline{\overline{\gamma}}} + \sqrt{1+\overline{\overline{\gamma}}}\right)}. \tag{7b}$$

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[1] The pdf of the SNR γ for a Rayleigh distributed signal envelope is

$$f(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}, \quad \gamma \geqslant 0 \tag{5}$$

where $\overline{\gamma}$ is average SNR; $\overline{\gamma} \triangleq \frac{\varepsilon_b}{N_0} E[r^2]$ and $E[r^2] = \sigma^2 \left(2 - \frac{\pi}{2}\right)$.

[2] If $f(X)$ is a convex function and X is a pdf, then $E\left[f\left(X\right)\right] \geqslant f\left(E\left[X\right]\right)$.

[3] If $\text{Re}(b) > 0$ and $\text{Re}(c) > 0$ then

$$\int_0^\infty e^{(a-b)t} \text{erfc}\left(\sqrt{at} + \sqrt{\frac{c}{t}}\right) dt = \frac{e^{-2\left(\sqrt{ac}+\sqrt{bc}\right)}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)},.$$

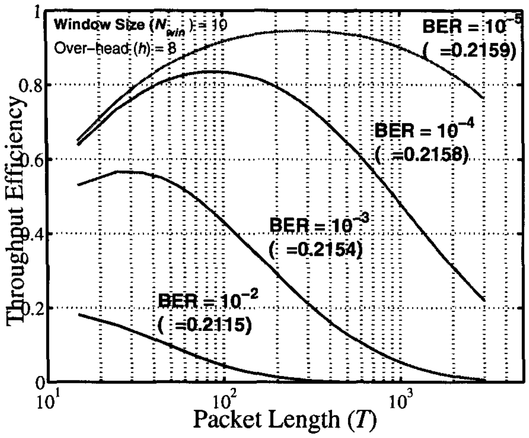

Fig. 1. Throughput Efficiency η for GBN with respect to data packet length T at different BERs; different fading σ.

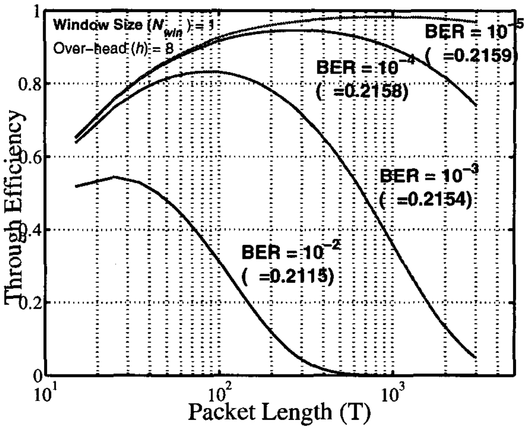

Fig. 2. Throughput efficiency η for SR as a function of the packet length at different BERs; different fading σ.

To achieve maximum throughput efficiency, the packet length should be optimum. In order to get the optimum packet length, we optimize the equation (2) with respect to the number of information bits per packet, T. In other word, we try to solve: $\frac{\partial \eta}{\partial T} = 0$. Substituting (6) in (2), and solving for $\frac{\partial \eta}{\partial T} = 0$ we get

$$\eta = \frac{T}{h+T} \cdot \frac{(1-p)^{h+T}}{(1-p)^{h+T} + \left(1-(1-p)^{h+T}\right) N} \tag{8}$$

where $N \triangleq N_{\text{WIN}}$. Differentiating the result with respect to T, we get

$$\begin{aligned} \frac{\partial \eta}{\partial T} &= \frac{h}{(h+T)^2} \cdot \frac{(1-p)^{h+T}}{(1-p)^{h+T} + \left(1-(1-p)^{h+T}\right) N} \\ &+ \frac{T}{h+T} \frac{N(1-p)^{h+T} \ln(1-p)}{\left((1-p)^{h+T} + \left(1-(1-p)^{h+T}\right) N\right)^2}. \end{aligned} \tag{9}$$

Finding the closed-form solution for the optimal packet length from (9) or the solution of $\frac{\partial \eta}{\partial T} = 0$ is not trivial; however, an iterative analytical solution is carried out to find the optimal packet length. Rewriting (9) for a polynomial form on one side, we get,

$$T + \frac{T^2}{h} = -\frac{(1-p)^{h+T}(1-N) + N}{N \cdot \ln(1-p)}. \tag{10}$$

This equation has no closed-form solution; therefore, an iterative approach for the solution of (10) is considered. An error between two successive iterations; $(k-1)^{\text{th}}$ and k^{th} iterations is defined as follows,

$$T_k = T_{k-1} + \Delta_k, \quad k = 1, 2, \cdots \tag{11}$$

where $T_0 = 1000$. In order to calculate Δ_k, we substitute T by $T_k = T_{k-1} + \Delta_k$ in the right side of (10). In the left side, substitute T by the a priori value of T_{k-1}, and then solve the resulting quadratic equation for the unknown Δ_k, i.e.,

$$T_{k-1} + \frac{T_{k-1}^2}{h} = -\frac{(1-p)^{h+T_{k-1}}(1-N) + N}{N \cdot \ln(1-p)} \tag{12a}$$

$$T_k + \frac{T_k^2}{h} = -\frac{(1-p)^{h+T_k}(1-N) + N}{N \cdot \ln(1-p)}. \tag{12b}$$

Putting (11) in (12b), using (12a) and the fact that $(1-p)^{\Omega} \approx 1 + (\Omega \ln(1-p)) + \frac{(\Omega \ln(1-p))^2}{2!}$, and after some manipulations, Δ_k is calculated as

$$\Delta_k = \frac{Nh + 2NT_{k-1} + (1-p)^{h+T_{k-1}}(1-N)h}{N + (1-p)^{h+T_{k-1}}(1-N)\left(\ln(1-p)\right)h/2}. \tag{13}$$

Substituting (13) in (11), the iterative equation to solve the optimum packet size, T_k, will be

$$T_k = T_{k-1} + \frac{Nh + 2NT_{k-1} + (1-p)^{h+T_{k-1}}(1-N)h}{N + (1-p)^{h+T_{k-1}}(1-N)\left(\ln(1-p)\right)h/2}. \tag{14}$$

Testing the convergence of this numerical algorithm using equation (10), it can be concluded that this iterative procedure easily converges, since the left hand side of this equation follows a second order polynomial (quadratic) form and the right hand side of that follows a power form. Depending on the value of N, algorithm usually converges in about 5-8 iterations for the error of less than 1%.

The throughput efficiency of a SR protocol can be found from (2). SR is a special case of GBN when window size is equal to one, i.e., $N = 1$. Using $N = 1$ in (8), we get,

$$\eta = \frac{T}{h+T} \cdot (1-p)^{h+T} \tag{15}$$

In order to find the optimum packet size, we differentiate this equation with respect to T. Solving the result for T_{opt}, we get

$$T_{\text{opt}} = \frac{-h + \sqrt{h^2 - \frac{4h}{\ln(1-p)}}}{2}. \tag{16}$$

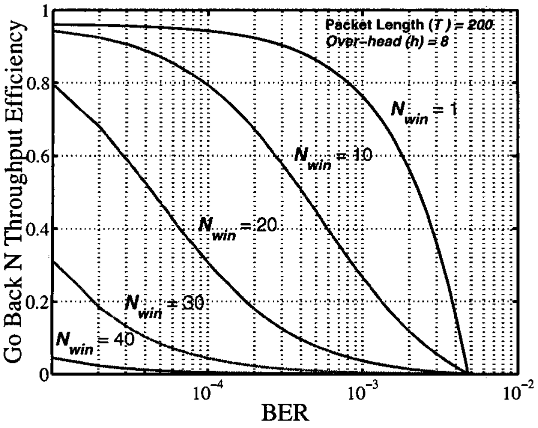

Fig. 3. Throughput Efficiency for GBN with respect to BER or channel fading σ, at different window sizes.

This result is consistent with the literature [10], [11]. We use this direct derivation in some numerical evaluations for the SR protocol.

IV-B. Results and Discussions

Figure 1 and Figure 2 show the throughput efficiency for GBN and SR, consecutively, as a function of data packet length for fixed window size, N, fixed overhead, h, and different values of fixed BER per graph. The figures show that for small packet length, the efficiency is low. This is because of the large overhead size with respect to the payload size. However, as the packet length increases, the throughput efficiency improves until it reaches a maximum value which is the optimal packet size. If the packet size becomes considerably large, the efficiency decreases because of the retransmissions, which are triggered by the packets arriving in error or lost in error; therefore, the optimal packet size increases with better channel conditions and reasonable size of payload with respect to overhead.

Figure 3 demonstrates the throughput efficiency as a function of channel BER for fixed overhead, fixed packet length, and different values of fixed window size per graph. As the window size increases, the efficiency decreases because any packet in error will result in sending all the subsequent packets sent prior to it, which reduces the throughput efficiency.

Figure 4 shows the optimal packet length as a function of the BER of the channel. The optimal packet length increases as the BER decreases. A lower BER allows transmission with larger packet size. Comparing the two protocols, the performance of SR is better than the performance of GBN. Since SR protocol retransmits the packets in error or the lost packets only, it improves the efficiency of the throughput and allows larger packet size under the same channel conditions. Figure 4 shows that the optimal packet length for the SR protocol is double that for the GBN under good channel conditions, (i.e., BER$\leqslant 2 \times 10^{-3}$); however, when the BER is large, (i.e, BER$\geqslant 2 \times 10^{-2}$), the performance of both is the same.

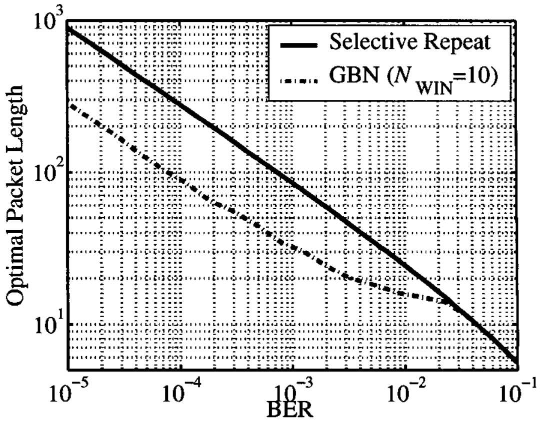

Fig. 4. The optimal packet length as a function of the channel BER or channel fading from (6).

V. CONCLUSIONS

The throughput efficiency improves as the packet size reaches an optimum value. For small packet size, the efficiency is low because of the overhead of the header of the packet. For large packet size, the efficiency decreases because of the retransmissions triggered by the packets arriving in error or lost due to fading. The SR protocol has better throughput efficiency than GBN and its optimal packet size is double the GBN optimal packet size under considerably good channel conditions.

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