On the Efficiency of Directional Antennas in MIMO Communication Systems

Omid Darvishi and Bahman Abolhassani
Department of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran
Omid_Darvishi@ee.iust.ac.ir, Abolhassani@iust.ac.ir

Hamidreza Saligheh Rad
Laboratory for Structural NMR Imaging
University of Pennsylvania
Philadelphia, PA 19104, USA
Hamid.Saligheh@gmail.com

Abstract— **It is conventionally believed that directional antennas are not as efficient as omnidirectional antennas in MIMO communications since they limit the effective angular spread of the scattering environment. However, it is often neglected that the power gain provided by directional antennas can compensate for the deleterious effect of their limited beamwidth. In this paper, we analyze the efficiency of directional antennas to investigate whether these antennas can improve the capacity of MIMO communication systems versus commonly used omnidirectional antennas. Employing a realistic MIMO channel model, we show that the performance of directional antennas depends on the scattering environment, the signal to noise ratio and the array configuration. Our simulation results suggest that in certain conditions directional antennas can indeed improve the capacity of MIMO systems compared to the capacity achieved by using omnidirectional antennas.**

I. Introduction

Since the pioneer works of [1] and [2], Multiple Input Multiple Output (MIMO) theory and techniques have enjoyed lots of progress. Promising advantages, provided by MIMO systems, has convinced standardization bodies to adopt MIMO technology in the future generation of communication systems. While the technology is getting mature, deployment issues are receiving more attention. As part of such issues, in this paper we are going to illustrate the effect of antenna directivity on the capacity of MIMO systems.

Investigation of the impact of directional antennas on the performance of the MIMO systems is relatively rare in the literature and the impact of antenna patterns is often excluded in modeling MIMO wireless channels. (See e.g. [3]–[5].) Besides mathematical simplicity of such consideration, it seems that there is confusion in the literature about the applicability of directional antennas in MIMO communications. As a matter of fact, directional antennas are usually employed in line-of-sight communications, whereas the exceptional capability of MIMO systems resides in the richness of the underlying scattering environment. Therefore it is sometimes believed that directional antennas are not good for MIMO systems. This belief stems from the fact that directional antennas have a limited beamwidth which restricts the effective angular spread of the propagation environment. The less the angular spread, as seen by the antennas is, the more the correlation between adjacent antennas will be, since less multipath components can be captured [3]. It is well known that correlation has always a detrimental effect on the capacity of MIMO systems with independent identically distributed (i.i.d.) inputs [6]. However it is important to note that the impact of directional antennas on the performance of MIMO systems is twofold:

a) Their beamwidth limits the effective angular spread which causes an increase in the correlation.

b) Their power gain enhances the received power in non-isotropic scattering environments which boosts the receiver signal to noise ratio (SNR).

The latter effect is often neglected perhaps due to a power normalization that is usually applied to ensure a fixed average SNR at the receiver. For a fair comparison of the performance of different MIMO systems, the channel matrix is scaled, such that the power transfer of the channel becomes a fixed constant [7]. Assuming the same transmit power for all systems under comparison, this allows to fairly compare those systems in the same receive SNR. However, as will be discussed later, conventional normalization schemes, result in systematically eliminating any constant multiplier that is constant over all subchannels from the channel matrix, including the antenna directivities.

Performance evaluation of directional antennas, needs two prerequisites, first an antenna-inclusive channel model that takes into account the pattern of antennas and second, a new normalization scheme that preserves the effect of antenna directivities. Considering both counteracting effect of directional antennas on the performance, it motivates us to search for the conditions that using directional antennas may act in favor of the performance. There are a few experimental measurements, supporting the idea that application of directional antennas in MIMO communications is beneficial [8][9]. However these measurement campaigns have been carried out in certain conditions, allowing for investigation of the impact of a few parameters on the capacity. To the best of our knowledge there is no thorough investigation of the performance of directional antennas, in the literature, considering the impact of all counteracting parameters under the same frame work. In this paper, by adopting a directional model of the MIMO channel together with a novel normalization scheme, we are trying to find the situations under which directional antennas are helpful.

The rest of this paper is organized as follows. Section II introduces the fading channel model used in this work. Section III presents a new normalization scheme that meets the conditions, mentioned above. Section IV investigates if directional antennas can improve the capacity performance of MIMO communication systems through simulations. Finally, this work is concluded in Section V with some guidelines for choosing suitable antennas in different situations.

II. MIMO Channel Model

We consider a single-user narrowband MIMO communication system, with N_t transmit and N_r receive antennas, communicating over a Rayleigh fading channel. Such a system admits the following input-output relationship

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is the channel matrix, $\mathbf{x}$ and $\mathbf{y}$ are the transmitted and received signal vectors and $\mathbf{n}$ represents additive white Gaussian noise vector at the receiver. Let's denote the correlation matrix of the elements of $\mathbf{H}$ by $\mathbf{R}$ such that $\mathbf{R} = \mathbf{E}[\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\dagger}]$, where $\mathrm{vec}(\cdot)$ is vectorizing operator and † denotes complex conjugate transposition. When the channel follows a Kronecker model, i.e. correlation between each two elements of $\mathbf{H}$ can be assumed separable as a multiplication of the transmit and receive correlations, $\mathbf{R}$ can be written as

$$\mathbf{R} = \mathbf{R}_t{}^T \otimes \mathbf{R}_r \tag{2}$$

Here $\mathbf{R}_t$ and $\mathbf{R}_r$ are fading correlation matrices at the receiver and transmitter respectively. In this case, the random channel $\mathbf{H}$ can be represented by

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{3}$$

where $\mathbf{H}_w$ is an $N_r \times N_t$ matrix whose entries are i.i.d. complex Gaussian random variables of zero mean and unit variance.

In order to relate the statistics of the channel matrix to the physical properties of the underlying channel, we employ a space-time-frequency cross-correlation function (STF-CCF) proposed in [10]. The STF-CCF represents the correlation in terms of distribution of propagation directions, antenna propagation patterns, array configuration and the mobile station velocity. According to the STF-CCF, the correlation $\rho^{mp,nq}$ between two sub-channels from transmit antenna p to receive antenna m, in time t_1 and frequency ω_1 and from transmit antenna q to receive antenna n, in time t_2 and frequency ω_2 is given by (4) at the bottom of this page. In (4), P_0 is a constant, $\Phi_\tau^{(\eta)}(s)$ is the η^{th}-order integration of the characteristic function of the propagation channel delay profile, η is the path-loss exponent, * stands for linear convolution and the superscript * denotes complex conjugate. $\mathcal{G}_{p,k}^B$ and $\mathcal{G}_{m,k}^M$ represent Fourier Series Coefficients (FSC) of the pattern of antennas at the Base Station (BS) and Mobile Station (MS) respectively. Also $\mathcal{F}_k^B, \mathcal{F}_k^M$ represent respectively FSC's of the power-azimuth spectrum (PAS) at BS and MS, i.e. the distribution of direction of departures (DOD) and direction of arrivals (DOA). STF separation vector $\mathbf{d}$ is defined at BS and MS as follows

$$\begin{aligned} \mathbf{d}_{p,q}^B &\triangleq \omega_1\mathbf{a}_p^B - \omega_2\mathbf{a}_q^B \\ \mathbf{d}_{\mathrm{m,n}}^{\mathrm{M}} &\triangleq \omega_1(\mathbf{a}_m^M - t_1\mathbf{v}) - \omega_2(\mathbf{a}_n^M - t_2\mathbf{v}) \end{aligned} \tag{5}$$

where $\mathbf{v}$ is the mobile velocity. Moreover function $\mathcal{W}(\mathbf{d},\mathcal{H}_k)$ has the following definition

$$\mathcal{W}(\mathbf{d},\mathcal{H}_k) \triangleq 2\pi \sum_{k=-\infty}^{+\infty} j^k e^{jk\angle d}\mathcal{H}_k(\omega)\,\mathrm{J}_k(\frac{|\mathbf{d}|}{c}) \tag{6}$$

in which $\mathrm{J}_k(\cdot)$ is the *k*-th order Bessel function of the first kind.

Although the STF-CCF is quite a general cross-correlation function, for simplicity we focus in the following on a narrowband single-carrier and stationary communication scenario. Specifically we put $\omega_1 = \omega_2 = \omega$ and $\mathbf{v} = 0$.

As it can be seen from (4), the correlation between two subchannels is a multiplication of three terms: the first term $P_0\Phi_\tau^{(\eta)}(j\omega_2 - j\omega_1)$, in single-carrier communication is a constant multiplier for all subchannels. It has no effect on the results and will be removed after channel normalization. The other two terms, each consists of a $\mathcal{W}(\cdot,\cdot)$ function, corresponding to correlation at the base station and mobile station. In a downlink scenario, these two functions build the correlation matrices at the transmitter and receiver, respectively.

III. Directivity-Preserving Normalization

A usual normalization method is to fix the average power transfer between each pair of transmit-receive antennas to unity, i.e.

$$\mathrm{E}[|h_{ij}|^2] = 1 \qquad i = 1,2,\ldots,N_r,\ j = 1,2,\ldots,N_t \tag{7}$$

where h_{ij} is the complex gain between the j^{th} transmit antenna and the i^{th} receive antenna. Allowing for an imbalance in the

$$\rho^{pm,qn}(t_1,t_2;\omega_1,\omega_2) = P_0\Phi_\tau^{(\eta)}(j\omega_2 - j\omega_1).\mathcal{W}(\mathbf{d}_{p,q}^B, \mathcal{G}_{p,k}^B(\omega_1) * \mathcal{G}_{q,-k}^B{}^*(\omega_2) * \mathcal{F}_k^B).\mathcal{W}(\mathbf{d}_{m,n}^M, \mathcal{G}_{m,k}^M(\omega_1) * \mathcal{G}_{n,-k}^M{}^*(\omega_2) * \mathcal{F}_k^M) \tag{4}$$

gain of subchannels, (7) can be generalized as

$$\mathrm{E}[\|\mathbf{H}\|_F^2] = \mathrm{tr}(\mathbf{R}) = N_t N_r \tag{8}$$

where $\|\mathbf{H}\|_F^2 = \sum_{i,j} |h_{ij}|^2$ is the squared Frobenius norm of the channel matrix $\mathbf{H} = [h_{ij}]$ and $\mathrm{tr}(\mathbf{R})$ is trace of the correlation matrix $\mathbf{R}$.

Conventional power normalization schemes, such as those in (7) and (8), mimic the function of some kind of power control by fixing the power transfer gain of the channel. This maintains a fixed average received power and thus removes the impact of antenna power gains from the channel matrix [7]. The reason is that, these schemes use a function of the channel itself as the normalizing factor. In this paper we suggest to normalize the power of the channel, using a reference channel. This reference channel is obtained by using omnidirectional antennas in the same environment as one, the directional antennas are going to work. We call such a normalization scheme, *directivity-preserving normalization scheme*. It is noted that a similar approach has been adopted in [9].

Let's denote the reference channel matrix by $\mathbf{H}^{ref}$ and its associated transmit and receive correlation matrices by $\mathbf{R}_t^{ref}$ and $\mathbf{R}_r^{ref}$. Now we define the normalization factors at the transmitter and at the receiver, respectively by

$$\eta_t = \frac{\mathrm{tr}(\mathbf{R}_t^{ref})}{N_t} \tag{9}$$

$$\eta_r = \frac{\mathrm{tr}(\mathbf{R}_r^{ref})}{N_r} \tag{10}$$

Using (9) and (10), the normalized correlation matrices are computed as

$$\mathbf{R}_t^{norm} = \frac{1}{\eta_t}\mathbf{R}_t \tag{11}$$

$$\mathbf{R}_r^{norm} = \frac{1}{\eta_r}\mathbf{R}_r \tag{12}$$

It is clear that applying directivity-preserving normalization, the normalized correlation matrices of the reference channel satisfy $\mathrm{tr}(\mathbf{R}_r^{ref,norm}) = N_r$ and $\mathrm{tr}(\mathbf{R}_t^{ref,norm}) = N_t$. Equivalently, for the normalized reference channel matrix, $\mathrm{E}[\|\mathbf{H}^{ref,norm}\|_F^2] = \mathrm{tr}(\mathbf{R}^{ref,norm}) = N_t N_r$. Therefore the average power transfer of the reference channel becomes constant, independent of the environment. However the transfer gain of the actual channel is varying dependent on the antenna types used at the transmitter and receiver.

IV. Capacity Performance of Directional Antennas

Assuming perfect channel state information at the receiver, the ergodic capacity of a MIMO channel with i.i.d. input is given by

$$C = \mathrm{E}\left[\log_2[\det(\mathbf{I}_{N_r} + \frac{\mathrm{SNR}}{N_t}\mathbf{H}\mathbf{H}^{\dagger})]\right] \tag{13}$$

where $\mathbf{I}_{N_r}$ is the identity matrix of the size $N_r \times N_r$ and $\mathrm{SNR} = P_t/\sigma_n^2$ is the transmit signal to noise ratio, with P_t and σ_n^2 being the total transmit power and the noise power, respectively. The expectation is with respect to the random channel. Inserting normalized correlation matrices of (11) and (12) into (3) to obtain realizations of the channel matrix, one can find the capacity of the channel through Monte Carlo evaluation of (13).

In order to access the capacity performance of directional antennas, we consider the downlink of a point-to-point narrowband MIMO system, operating at the frequency of 1 GHz. We assume the 2D scattering environment can be represented by a *truncated Laplacian* PAS, given by

$$f(\theta) = \begin{cases} \dfrac{1}{2\,\mathrm{a}(1-e^{-\frac{\pi}{\mathrm{a}}})} e^{-\frac{|\theta-\theta_m|}{\mathrm{a}}} & \theta_m - \pi < \theta \le \theta_m + \pi \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where θ represents either the DODs or DOAs and θ_m is their corresponding mean. "a" is a parameter that controls the root mean square (rms) angular spread of the propagation environment, i.e. the standard deviation of θ. We denote this parameter at the transmitter and the receiver by $\mathrm{a_t}$ and $\mathrm{a_r}$, respectively.

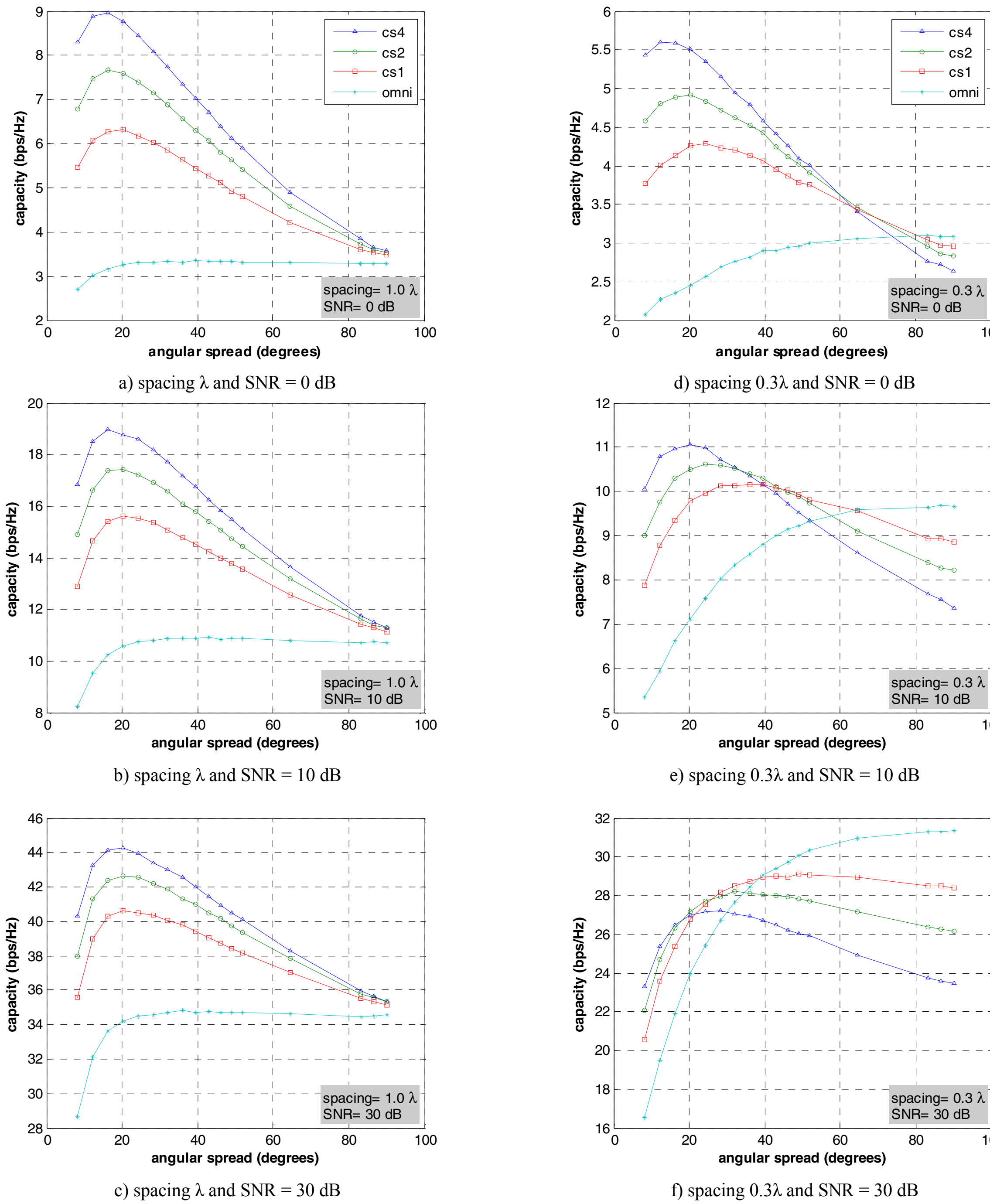
For investigating the impact of directionality of antennas on the capacity of MIMO systems, we employ antennas with a field pattern of $G_n(\theta) = \sqrt{d_n}\cos^n\theta$, where d_n is the antenna directivity. TABLE I. summarizes the directivity and half power-beamwidth (HPBW) of this type of antenna for $n = 1,2,3,4$. The directivities are with respect to that of omnidirectional antenna in dBi and HPBWs are in degrees.

For the sake of simplicity, we assume that both transmitter and receiver are equipped with uniform linear arrays (ULA) of antennas of the same type. Furthermore in the illustrations, it is assumed that antenna spacing at both sides is the same and transmitter and receiver, each has four antennas.

Fig. 1 demonstrates the performance of directional antennas compared with that of omnidirectional antenna, where capacity has been plotted as a function of the environment angular spread. There we use "*CS1*", "*CS2*", "*CS4*" and "*omni*" to denote antennas with pattern of $G_n(\theta)$ for n = 1, 2, 4 and omnidirectional antenna, respectively. For the sub-figures on the left column, antennas are a wavelength (λ) apart while the antenna spacing is 0.3λ in the right-side sub-figures.

TABLE I. Properties of the Antennas in the Study

antenna type	directivity (dBi)	HPBW
n=1 (CS1)	3	90
n=2 (CS2)	4.3	65.5
n=3 (CS3)	5	54
n=4 (CS4)	5.6	47



a) spacing λ and SNR = 0 dB

d) spacing 0.3λ and SNR = 0 dB

b) spacing λ and SNR = 10 dB

e) spacing 0.3λ and SNR = 10 dB

c) spacing λ and SNR = 30 dB

f) spacing 0.3λ and SNR = 30 dB

Figure. 1 Comparison of the capacity performance of omnidirectional and directional antenna

For each of the two spacing, the capacity has been plotted in three different SNRs, 0dB, 10dB and 30dB. As it can be seen from sub-figures (a), (b) and (c) when the spacing between antennas is not an issue and antenna can be placed about a wavelength apart, directional antennas in the study almost always outperform omnidirectional antenna.

However as the angular spread increases the superiority of directional antennas diminishes. In near isotropic environments[1] all types of antennas receive the same power. In such situations the directionality of antennas can't help anymore, meanwhile the environment is rich enough to decorrelate antennas in spite of their confined beamwidth. Thus, all the antennas in the study perform similarly in rich scattering environments. It is noted that antenna directionality

[1] In the isotropic scattering environment (rms) angular spread is 104 degrees.

is favorable to the extent that its associated narrow beamwidth does not negate its power gain by highly decreasing the effective angular spread.

On the other hand, the figures on the right column, (d), (e) and (f), show an interesting phenomenon. In case of compact arrays, where the antennas should be placed near to each other (here 0.3λ) because of the space limitations, the performance is dependent on the SNR and antenna spacing as well as the angular spread.

In poor scattering environments, the uniform radiation of the omnidirectional antennas is of little help and the limiting factor is the restricted angular spread of the environment. In such a situation, the rather small beamwidth of directional antennas does not add so much to the correlation of the adjacent antennas while their directivity boost the receive SNR which gives them a superiority over the omnidirectional antennas. This effect is more distinguishable in low SNRs.

On the contrary, in rich scattering environments, the power gain of directional antennas is no more helpful, since the signal power is dispersed over a large extent of directions. On the other hand the limited beamwidth of directional antennas cause a decrease in the effective angular spread which severely increases the correlation. Therefore omnidirectional antennas outperform in these situations.

These results suggests the following general guidelines for selection of antenna type

- For widely spaced antenna arrays, directional antennas can be beneficial in a wide range of SNRs and different environments with various amount of scattering.
- For size-constrained arrays, in rich scattering environments, omnidirectional antennas perform better, but in poor scattering environments directional antennas have a better performance. In the environments with an angular spread comparable to the beamwidth of available directional antennas, operating SNR determines which antenna is more suitable.

The author of [10], as a guideline for antenna design, states that it is desirable to use antennas with patterns that are well-matched to the environment PAS. Our results show that in choosing the suitable antenna, operating SNR and antenna spacing are other factors that should also be considered together with the PAS.

It is emphasized that in our study we have assumed that the transmitter/receiver can estimate the mean DOD/DOA accurately and direct the antenna beams towards them. Besides, the transmit/receive arrays are broadside to the mean DOD/DOA. For omnidirectional antennas, broadside array orientation is shown to achieve the lowest inter-element correlation [12]. However, in case of directional antennas, whether such orientations of the antenna beams and the arrays are optimal or how much a mismatch between actual and optimum orientations can degrade the performance is open for further investigations.

V. Conclusion

In this paper, we investigated the performance of directional antennas in MIMO communications. We revisited the common belief about the inefficiency of directional antennas and looked into the reasons behind this belief. We argued that directional antennas can improve the performance when their power gain dominates the adverse effect of correlation increase caused by their limited beamwidth. Conventional power normalization schemes fail to consider this power gain and it might be the reason why people often believe directional antennas are inefficient for MIMO systems. To overcome this shortcoming, we presented a new normalization scheme that preserves the gain of antennas while provides a fair comparison of the different antennas. The results show that choice of a suitable antenna in the context of capacity improvement depends on the array volume, the angular spread of the environment and the operating SNR. Finally we provided some general guidelines for antenna selection.

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