JOINT ESTIMATION AND DETECTION FOR TRANSMIT DIVERSITY OVER FAST FADING CHANNELS

*Saeed Gazor, and Hamidreza Saligheh Rad*

saeed.gazor@ece.queensu.ca,radh@ee.queensu.ca
Department of Electrical and Computer Engineering,
Queen's University, Kingston, Ontario, K7L 3N6, Canada.

*Abstract–*

This paper presents a simple channel estimation algorithm based on the Maximum *A Posteriori* (MAP) method for a Dual Transmit Diversity system, using a Space-Time (ST) receiver that takes into account the channel estimation error and assumes that the unknown channel has a given complex bivariate Gaussian probability density function (pdf) (*i.e.*, a Ricean distribution). The channel pdf is updated for the next iteration by estimates of the statistics of the channel coefficients, and a very simple adaptive algorithm is derived for channel estimation. Simulations confirm that the proposed scheme achieves robust performance in the time-varying scenario with few training symbols, when the Signal-to-Noise Ratio (SNR) is not very low. The algorithm is capable of efficiently tracking a fast Raleigh fading channel. However, the occurrence of two types of ambiguities might result in error propagation which are addressed here.

## I. INTRODUCTION

There is a growing need for high-quality wireless communications over fast multipath fading channels. In most scattering scenarios, antenna diversity is a practical and efficient technique for reducing the effect of multipath fading [1]. These schemes employ pre-coding, namely Space-Time Coding (STC), which is appropriate for multiple transmit antenna systems. STC leads to a considerable increase in bandwidth efficiency and system capacity [2].

Although it has been proved in [5] that in the presence of small errors in channel state information, STCs still result in an improved bandwidth efficiency over classical transmitting schemes, considerable degradation is observed when the channel estimation error increases. This could be improved by sending more pilot symbols (training symbols) during the transmission at the cost of losing some bandwidth efficiency, especially in the case of fast time-varying channels. The performance of existing receivers for STC methods degrades dramatically in time-varying channels [3], and an efficient channel tracking algorithm is required to provide the CSI. Hence, robust detection algorithms for these methods are needed for good operation when the CSI is not exactly known.

In this paper, a simple algorithm is developed for the channel estimation, employing the MAP data detection algorithm proposed in [12] that takes into account the channel estimation errors. This estimator updates the parameters of the channel distribution in each iteration, which in turn is used in the next iteration for both data detection and channel estimation. Results indicate that the proposed algorithm requires only a few training symbols, *e.g.*, one block of symbols, for initialization. Simulations verify the robust performance of the proposed receiver for time-varying channels.

The paper is organized as follows: In Section II the system and receiver structure are provided. In Section III, a Baysian estimator is presented for the channel parameters and its performance is studied in a special case. Permutation ambiguity is also proposed in this section. Finally, some concluding remarks are discussed in Section IV.

## II. SYSTEM MODEL AND RECEIVER STRUCTURE

For simplicity in this paper, the Dual Transmit Diversity (DTD) technique is considered [1]. This scheme can be described as follows:

$$R_k \triangleq \begin{bmatrix} s_{1,k} & s_{2,k} \\ -s_{2,k}^* & s_{1,k}^* \end{bmatrix} \begin{bmatrix} h_{1,k} \\ h_{2,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \end{bmatrix} = S_k H_k + N_k. \quad (1)$$

where $R_k \in \mathbb{C}^{2L}$ and $N_k \in \mathbb{C}^{2L}$ are the received signal and the Additive White Gaussian Noise (AWGN), respectively. Transmitted symbols, $s_{1,k}$ and $s_{1,k}$, both take their values randomly from $C = \{c_i \in \mathbb{C}^L\}_{i=1}^{K}$, where $K$ is the number of constellation points and $L$ is the dimension of the transmitted signal space. The channel gains, $h_{1,k}$ and $h_{2,k}$, are complex random variables. The notations $(.)^*$ and $(.)^H$ stand for complex conjugate and Hermitian, respectively.

We use the detector proposed in [12] that takes into account the uncertainties of CSI and adopt the notations $(.)_{k|k-1}$ and $(.)_{k|k}$ to denote *a priori* estimates and *a posteriori* estimates at the $k^{\text{th}}$ iteration, respectively. Following these notations, Table I summarizes the proposed detection algorithm. The receiver is provided with the received signal $R_k$ and inaccurate *a priori* channel information, $H_{k|k-1}$, where the matrix, $\Sigma_{k|k-1}$, represents *a priori* covariance of the

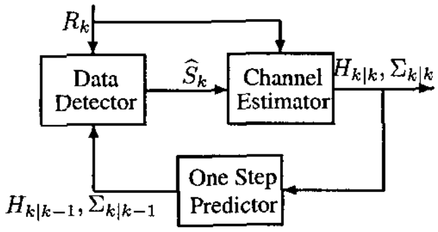


Fig. 1. The block diagram of the joint Detector and Estimator, where $H_{k|k}$ and $\Sigma_{k|k}$ represent the estimated CSI.

channel errors. The value $\Sigma_{k|k}$, represents the *a posteriori* covariance of the channel coefficients when the error probability is very small, satisfies following inequality [12]:

$$\text{Cov}\Big(H_{k|k}|R_k, S_k = \widehat{S}_k\Big) = \Sigma_{k|k} \leqslant \Sigma_{k|k-1}. \quad (2)$$

Table I and the above inequality show that $H_{k|k}$ can be considered as the *a posteriori* estimate of the channel when the Symbol Error Probability (SEP) is low. This implies that $H_{k|k}$ can be used as the output of an iterative algorithm to estimate the channel coefficients.

## III. Joint Channel Estimation and Detection

The adaptive channel estimation is based on the concept of decision feedback. In each iteration, the *a posteriori* pdf is first maximized by the detector proposed in Section II. Then the channel *a posteriori* pdf is maximized to estimate the channel. This channel estimator characterizes the *a posteriori* pdf of the channel parameters as a complex Gaussian pdf. The structure of the algorithm is shown in Figure 1. The output of the estimator is used in the feedback loop via a one-step predictor for detection in the next time iteration. (2) validates this approach, as the covariance matrix of the estimated channel vector decreases in every iteration, and the estimator uses an equation based on this covariance matrix. Following the above notation and the result of (2), the iterative channel estimation algorithm, which is designed based on the detection results, is summarized in Table II, where $\widehat{S}_k$ is the detected symbol matrix and $R_k$ is the received signal vector at the $k^{\text{th}}$ iteration. Figure 2 shows the block diagram of the proposed channel estimator, when the error probability is ignored.

After presenting performance curves of the estimator, the performance of the estimator in time-invariant and time-varying scenarios is also investigated. Simulation results show the improved capability of a diversity scheme using channel estimation. This also shows that transmission diversity, while providing better bandwidth efficiency [2, 5], enhances the capability of the channel estimation.

*Case 1: Time-Invariant Environment:* Figure 3 shows the evaluation of the estimator performance in one iteration. This estimator is evaluated for two different cases:

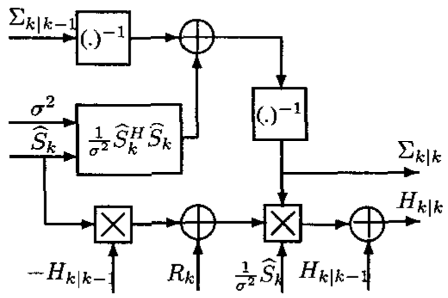


Fig. 2. The block diagram of the CSI estimator.

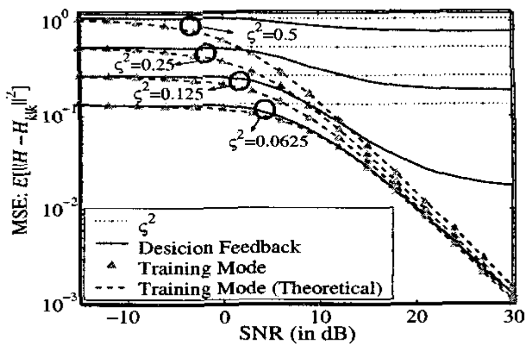


Fig. 3. Performance of the channel estimator in one iteration versus different $\varsigma^2$, and $H_{k|k-1} = [1; 1]$. The experiment is repeated 125000 times.

1) When $\widehat{S}_k$ is known, *i.e.*, $\widehat{S}_k$ is a training pair and there is no error probability in detection; and 2) When $\widehat{S}_k$ is obtained by the proposed detector in Table I. A comparison of the performance between these two cases shows a considerable error reduction just after one iteration, so the impact of the decision errors has no practical consequence if the channel estimation error, $\varsigma^2$, is small enough. Otherwise, a small number of training symbol pairs are required. Here, the SNR is defined by $10\log\left(\frac{|S|(2\varsigma^2+\|\bar{H}\|^2)}{2\sigma^2}\right)$.

*Case 2: Time-Variant Channel:* In order to derive an adaptive channel estimation algorithm, we assume that the channel variation, $\Delta_k \triangleq H_{k+1} - H_k$, is independent of $H_k$ (*i.e.*, a Markov model of the first order), and $\text{Cov}(\Delta_k) = \eta I$. This gives:

$$\Sigma_{k+1|k} \triangleq \text{Cov}\left(H_{k+1} - H_{k|k} \mid \widehat{S}_k = S_k\right),$$
$$= \eta I + \text{Cov}\left(H_k - H_{k|k} \mid \widehat{S}_k = S_k\right) = \eta I + \Sigma_{k|k}.$$

In the above, $\Delta_k$ is also assumed to be zero-mean and independent of $S_k$ and $\widehat{S}_k$. All these assumptions are reasonable in practice. By using this approach, the iterative LMS-type algorithm is proposed as in Table II, in which $\mu$ is the step-size introduced in the algorithm to control the trade-off between speed of convergence and maladjustment, and $\eta$ is a

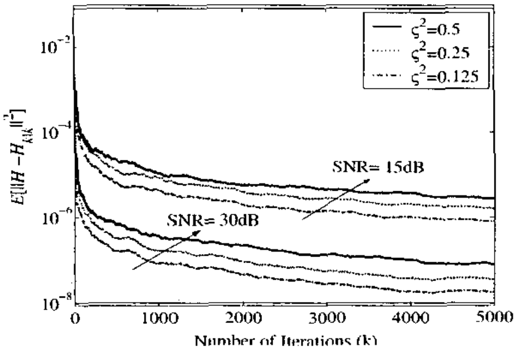


Fig. 4. Mean Square Error (MSE) of the proposed estimator, *i.e.*, $E\left[\|H - H_{k|k}\|^2\right]$, for different values of $\varsigma^2$ (average of 25 experiments; the training sequence is just one iteration).

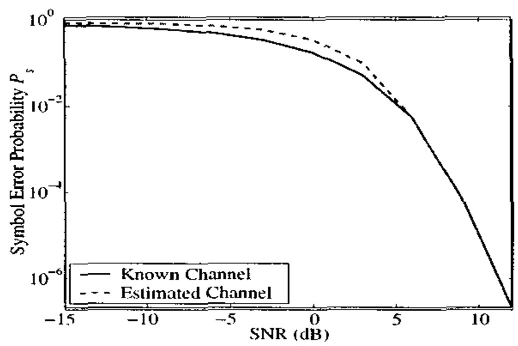


Fig. 5. SEP for time-invariant channel using; Dashed: proposed algorithm in steady state and one training pair; Solid: with channel information.

compensation factor for the channel variations in two successive samples. It is to be noted that the step-size, $\mu$, and the compensation factor, $\eta$, should be adjusted and finely tuned for each environment. The performance of this algorithm is evaluated first for one iteration. This evaluation is then expanded for two scenarios of time-invariant and time-varying channels in the next subsections.

### *A. Stationary Scenario*

For a time-invariant random channel, $H_k$, the learning ability of the channel estimation algorithm is investigated, employing a 4QAM modulation scheme. The time-invariant channel is generated by two correlated complex Gaussian random variables. The implementation of the proposed algorithm requires a short training period to initialize the *a priori* channel estimate. After this period, it switches to the decision-directed mode. During the training mode the receiver knows the transmitted symbols, while in the decision-directed mode, the detected symbols take their place. In the stationary case, simulations show that the algorithm needs just one pair of symbols (one iteration) for the training mode to recognize the direction of the channel gains, which is depicted in Figure 4 for different values of $\varsigma^2$ and SNRs. The results show that the estimation algorithm performs better when the SNR increases. Figure 5 presents the effect of the channel estimation error on the SEP with one pair of training symbols in the steady state mode. The result shows that the estimation algorithm has very good performance at high SNRs when compared with a receiver that knows the channel. However, some degradation is observed at low SNRs. The bandwidth cost for one training pair is not significant.

### *B. Time-Varying Scenario; Slow Fading Channel*

*1) Fading Channel Simulator:* In wireless mobile communications, a time-varying channel is caused mainly by Doppler shifts and the carrier offset frequency. These can be modeled either deterministically through basis function expansion or randomly as autoregressive (AR) processes [6]. Here, the simple Clark and Gans fading model (*i.e.*, $E[h_{i,k}h_{i,k+m}] = J_0(\omega_d m)$ where $J_0$ is the Bessel function) [8] based on Inverse Fast Fourier Transform (IFFT) [7] is used to simulate the time-varying channel parameters. For more details, see [11].

*2) Simulation Results:* Figure 6 shows the performance of the proposed estimator. Here, just a few training symbols are sent in the initialization mode to obtain good tracking accuracy. Afterwards, the algorithm distinguishes the channel variations and tries to track the channel vector. Note that in lower values of SNR, pilot symbols need to be sent periodically to achieve good tracking performance.

There are two types of ambiguities that might result in error propagation. The phase ambiguity, described in [12], can be resolved either by using differential modulation schemes or by periodic transmission of a few training symbols. Differential ST modulation schemes, *e.g.* [3, 4], obviate the channel phase ambiguity at the expense of 3dB SNR loss in the performance. The other ambiguity is the following:

**Permutation Ambiguity:** In (1), it is seen that the received signal, $R$, remains invariant if $(s_1, s_2, h_1, h_2)$ is transformed into $(s_2, -s_1, h_2, -h_1)$. Therefore, in practice in a non-stationary environment, if $h_2$ passes by $-h_1$, the receiver might detect $(s_2, -s_1)$ instead of $(s_1, s_2)$. In other words, the receiver ignores the crossover; therefore the channel estimation algorithm will track $(h_2, -h_1)$ instead of $(h_1, h_2)$. After crossover, algorithm fails to recognize the change of the situation and continues to consider $(s_2, -s_1, h_2, -h_1)$ instead of the quadruplet $(s_1, s_2, h_1, h_2)$. This phenomenon can be resolved 1) by the detection of crossovers and an improved tracking scheme for the channel vector, 2) by periodic use of one or two training symbols, 3) by the use of differential STC coding methods, *e.g.* [3, 4], or 4) by a combination of these methods. Figure 7 shows the effect of this permutation ambiguity on the channel tracking. As seen in this figure, this permutation mostly happens in low

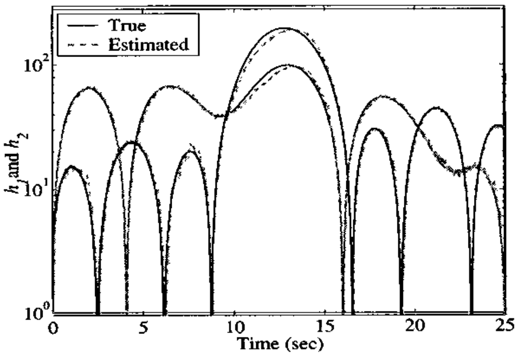


Fig. 6. Channel gains, $|h_1|$ and $|h_2|$, and their estimated values; SNR=15dB, a few number of training symbols, and $\omega_d$= 0.1 rad/sec.

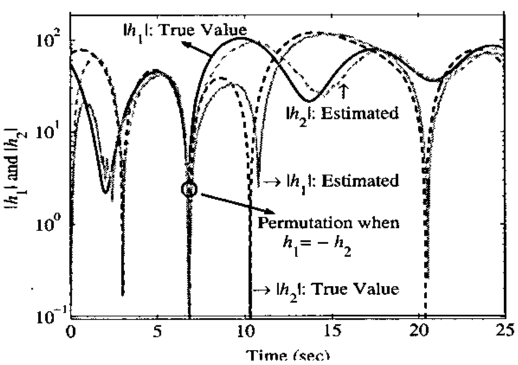


Fig. 7. Effect of permutation ambiguity on channel tracking performance; SNR=2dB, 10 training pairs, and $\omega_d$= 0.1 rad/sec.

SNRs when both (or one) channels are in deep fades. After the permutation event, the algorithm is unable to continue tracking correctly beyond the crossover point.

In [13], some remedies are suggested to combat these ambiguities. One method is an improved tracking algorithm based on an estimation of speed of variation of $H_k$ [9, 10]. This algorithm detects possible crossovers by comparing $|h_1 + h_2|$ with a small positive threshold $\epsilon > 0$, and if there is any possibility that a crossover has occured, the channel parameters are updated by using only the estimated speeds from previous iterations. For more details, see [13].

TABLE I. Summary of Detection Algorithm

$$
\begin{aligned}
H_{k|k} &= H_{k|k-1} + \tfrac{1}{\sigma^2}\Sigma_{k|k}C_{p,q}^H\left(R_k - C_{p,q}H_{k|k-1}\right),\\
M(C_{p,q}, R_k) &= \log|\Sigma_{k|k}| + H_{k|k}^H\Sigma_{k|k}^{-1}H_{k|k},\\
\widehat{S}_k &= \arg\max_{c_p,c_q\in C} M(C_{p,q}, R_k)\\
\Sigma_{k|k} &= \left(\alpha_{p,q}I + \Sigma_{k|k-1}^{-1}\right)^{-1},\\
\alpha_{p,q} &\triangleq \frac{\|c_p\|^2+\|c_q\|^2}{\sigma^2} = \frac{|C_{p,q}|}{\sigma^2},\quad C_{p,q} \triangleq \begin{bmatrix} c_p & c_q \\ -c_q^* & c_p^* \end{bmatrix}.
\end{aligned}
$$

TABLE II. Summary of Channel Estimation Algorithm

$$
\begin{aligned}
\widehat{S} &\triangleq \begin{bmatrix} \widehat{s}_{1,k} & \widehat{s}_{2,k} \\ -\widehat{s}_{2,k}^* & \widehat{s}_{1,k}^* \end{bmatrix} \leftarrow \text{from Table I},\\
H_{k|k} &= H_{k|k-1} + \mu\Sigma_{k|k}\widehat{S}_k^H\left(R_k - \widehat{S}_kH_{k|k-1}\right),\\
\Sigma_{k|k}^{-1} &= \tfrac{1}{\sigma^2}\widehat{S}_k^H\widehat{S}_k + \Sigma_{k|k-1}^{-1}.\\
\text{Prediction:}\quad H_{k+1|k} &= H_{k|k},\\
\Sigma_{k+1|k} &= \Sigma_{k|k} + \eta I.
\end{aligned}
$$

## IV. Conclusions

We show that exploiting diversity transmission in a fading environment not only results in a considerable improvement in SEP, but also greatly improves channel estimation and tracking. A very simple and efficient channel estimation algorithm based on MAP detection scheme is presented. The combination of the proposed detection and channel tracking algorithms using the decision feedback concept provides an efficient and successful solution in this context. Error propagation caused by two types of ambiguities is outlined.

## References


[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1451–1458, Nov. 1998.

[2] V. Tarokh, N. Seshadri, A. Calderbank, "Space-Time Codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744–765, March 1998.

[3] B.L. Hughes, "Differential Space-Time Modulation," *IEEE Transactions on Information Theory*, vol. 46, pp. 2567-2578, Nov. 2000.

[4] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE Journal on Selected Areas in Communications*, vol. 18, pp. 1168-1174, July 2000.

[5] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-Time Codes for high data rate wireless communication: Performance criterion in the presence of channel estimation errors, mobility and multipaths," *IEEE Transactions on Information Theory*, vol. 47, no. 2, Feb. 1999.

[6] W. Turin, R. Jana, C. Martin, and J. Winters, "Modeling Wireless Channel Fading," *54th IEEE Conference on Vehicular Technology*, vol. 3, no. 2, pp. 1740-1744, Fall 2001.

[7] M. F. Pop and N. C. Beaulieu, "Limitations of Sum-of-Sinusoids Fading Channel Simulators," *IEEE Transactions on Communications*, vol. TCOM-49, pp. 699-708, April 2001.

[8] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 1996.

[9] S. Gazor, "Prediction in LMS-type adaptive algorithms for smoothly time varying environments," *IEEE Transactions on Signal Processing*, vol. SP-47, no. 6, June 1999.

[10] S. Affes, S. Gazor and Y. Grenier, "An algorithm for multisource beamforming and multitarget tracking," *IEEE Transactions on Signal Processing*, vol. SP-44, no. 6, pp. 1512-1522, June 1996.

[11] S. Gazor and H. S. Rad, "Space-Time Coding Ambiguities in Joint Adaptive Channel Estimation and Detection," submitted to *IEEE Transactions on Signal Processing*, vol. SP-, April 2003.

[12] H. S. Rad and S. Gazor, "An Optimal Receiver for Transmission Diversity over Uncertain Channels," to be appeared in ICASSP'03, Hong Kong.

[13] H. S. Rad and S. Gazor, "Transmission Diversity Ambiguities and Adaptive Channel Tracking," to be appeared in ISSPA'03, France.